(b) states 2 and 02 are the actual and stagnation states of the fluid leaving the diffuser.

(4) The velocity coefficient C_v is defined:

$$C_v = \frac{\text{Actual velocity at nozzle exit}}{\text{Velocity at nozzle exit with isentropic flow and same exit pressure}}$$

REVIEW PROBLEMS

PROBLEM 1

Determine the final equilibrium state in English units when 2 lbm of saturated liquid mercury at 1 psia is mixed with 4 lbm of mercury vapor at 1 psia and 1,400°F. During the process the pressure in the cylinder is kept constant and no energy is lost between the cylinder and mercury.

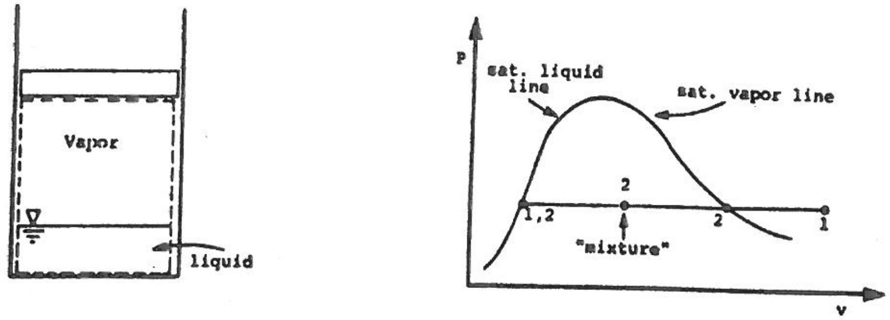


Figure 34. (a) The control mass

(b) The process representation

SOLUTION

Since the amount of liquid might change during the process, the liquid or only the vapor cannot be taken as the control mass. Instead, take the entire 6 lbm of mercury. By assumption, no energy transfer as heat occurs, but the volume is expected to change, resulting in an energy transfer as work. The only energy stored within the control mass is the internal energy of the mercury; the energy balance, made over the time for the process to take place, is therefore (Figures 34 and 35)

$$\underset{\substack{\text{energy}\\ \text{input}}}{W} = \underset{\substack{\text{increase in}\\ \text{energy storage}}}{\Delta U}$$

where $\Delta U = U_2 - U_1$

The work calculation is made easy by the fact that the pressure is constant. When the piston moves an amount *dx*, the energy transfer as work from the environment to the control mass is

$$dW = PAdx = -PdV.$$

Integrating,

$$W = \int_1^2 -PdV = P(V_1 - V_2).$$

Combining with the energy balance obtain

$$U_2 + PV_2 = U_1 + PV_1 \tag{1}$$

TABLE 3
PROPERTIES OF SATURATED MERCURY

		Enthalpy, Btu/lbm		
P, psia	**T,°F**	**Sat. liq.**	**Evap.**	**Sat. vap.**
0.010	233.57	6.668	127.732	134.400
0.020	259.88	7.532	127.614	135.146
0.030	276.22	8.068	127.540	135.608
0.050	297.97	8.778	127.442	136.220
0.100	329.73	9.814	127.300	137.114
0.200	364.25	10.936	127.144	138.080
0.300	385.92	11.639	127.047	138.086
0.400	401.98	12.159	126.975	139.134
0.500	415.00	12.568	126.916	139.484
0.600	425.82	12.929	126.868	139.797
0.800	443.50	13.500	126.788	140.288
1.00	457.72	13.959	126.724	140.683
2.00	504.93	15.476	126.512	141.988
3.00	535.25	16.439	126.377	142.816
5.00	575.70	17.741	126.193	143.934

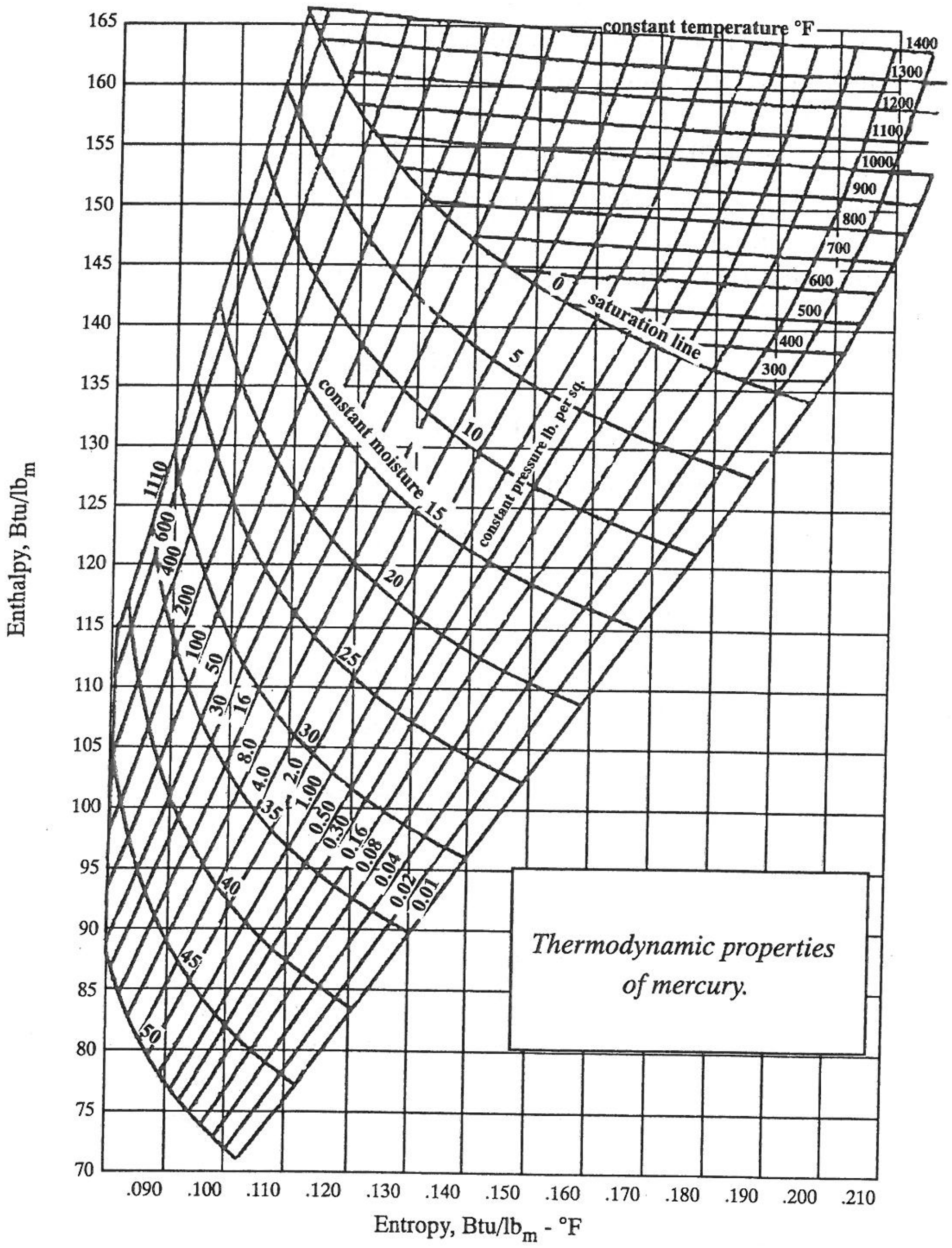


Figure 35. Thermodynamic properties of mercury

To evaluate the initial terms assume that the liquid is in an equilibrium state and the vapor is in an equilibrium state, even though they are not in equilibrium with one another. The graphical and tabular equations of state, Figure 35 and Table 3 for the thermodynamic properties of saturated mercury, may then be employed for each phase. Since the available equation-of-state information is in terms of the enthalpy property, express the right-hand side of equation (1) as

$$\begin{aligned} U_1 + PV_1 &= M_{l_1}u_{l_1} + M_{v_1}u_{v_1} + P\left(M_{l_1}v_{l_1} + M_{v_1}v_{v_1}\right) \\ &= M_{l_1}h_{l_1} + M_{v_1}h_{v_1} \end{aligned}$$

Now, from the tables, the initial liquid enthalpy is (saturated liquid at 1 psia)

$$\begin{aligned} h_{l_1} &= 13.96 \text{ Btu/lbm} \\ T_1 &= 457.7°\text{F} \end{aligned}$$

The initial vapor enthalpy is found from Figure 35 as

$$h_{v_1} = 164 \text{ Btu/lbm.}$$

Substituting the numbers,

$$U_1 + PV_1 = 2 \times 13.96 + 4 \times 164 = 684 \text{ Btu}.$$

The final state is a state of equilibrium, for which

$$U_2 + PV_2 = M(u + Pv)_2 = Mh_2.$$

The enthalpy in the final state is therefore

$$h_2 = \frac{684 \text{ Btu}}{6 \text{ lbm}} = 114 \text{ Btu / lbm}.$$

The final pressure and enthalpy may be used to fix the final state. Upon inspection of Figure 35, the final state is a mixture of saturated liquid and vapor at 1 psia and the "moisture" $(1 - x)$ is about 21 percent (0.79 quality). Alternatively, the information in Table 1, could have been used.

$$\begin{aligned} 114 &= \left(1 - x_2\right) \times 13.96 + x_2 \times 140.7 \\ x_2 &= 0.79 \end{aligned}$$

PROBLEM 2

The gauge pressure in an automobile tire when measured during winter at 32°F was 30 N/m^2. The same tire was used during the summer, and

and

$$Q = 0.5(2{,}804.8 - 1{,}444.6) + 91.0 = 771.1\ \text{kJ}$$

PROBLEM 4

Steam at 3 MPa, 300°C leaves the boiler and enters the high-pressure turbine (in a reheat cycle) and is expanded to 300 kPa. The steam is then reheated to 300°C and expanded in the second stage turbine to 10 kPA. What is the efficiency of the cycle if it is assumed to be internally reversible?

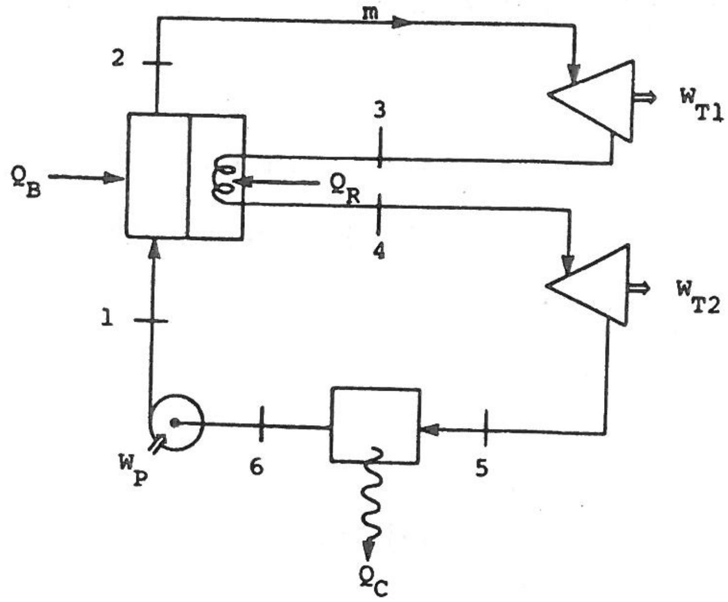


Figure 36. Schematic of heating cycle

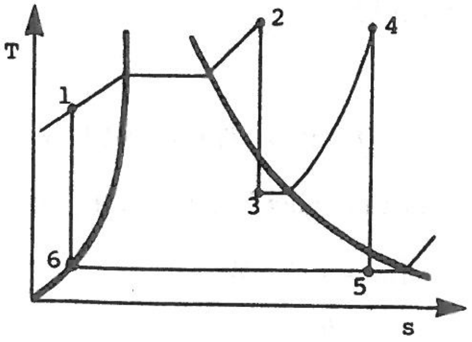


Figure 37. *T-s* diagram for heating cycle

SOLUTION

The efficiency η can be obtained from the following equation:

$$\eta = \frac{\dot{W}_{t_1} + \dot{W}_{t_2} - \dot{W}_p}{\dot{Q}_b - \dot{Q}_r} \tag{1}$$

To calculate $\dot{W}_{t_1}$ assume that the turbine is adiabatic and neglect kinetic and potential energy changes. Applying the first law to the turbine,

$$\dot{W}_{t_1} = \dot{m}\left(h_2 - h_3\right).$$

From the steam tables,

$$h_2 = 2{,}993.5 \text{ kJ/kg}$$
$$s_2 = 6.5390 \text{ kJ/kg} - \text{K}$$

To find h_3 for the internally reversible adiabatic process 2 → 3:

$$s_2 = s_3 = 6.5390 \text{ kJ/kg} - \text{K}$$

At state 3,

$$s_{f_3} = 1.6718 \text{ kJ/kg} - \text{K} \qquad h_{f_3} = 561.47 \text{ kJ/kg}$$
$$s_{fg_3} = 5.3201 \text{ kJ/kg} - \text{K} \qquad h_{fg_3} = 2{,}163.8 \text{ kJ/kg}$$
$$s_{g_3} = 6.9919 \text{ kJ/kg} - \text{K} \qquad h_{g_3} = 2{,}725.3 \text{ kJ/kg}$$

$$s_2 = s_3 = s_{f_3} + x_3 s_{fg_3}$$
$$6.5390 = 1.6718 + x_3(5.3201)$$
$$x_3 = 0.915$$
$$h_3 = h_{f_3} + x_3 h_{fg_3}$$
$$= 561.47 + 0.915(2{,}163.8)$$
$$= 2{,}542 \text{ kJ/kg}$$

$$\frac{\dot{W}_{t_1}}{\dot{m}} = h_2 - h_3$$
$$= 2{,}993.5 - 2542$$
$$= 452 \text{ kJ/kg}$$

Similarly, to find $\dot{W}_{t_2}$

$$\dot{W}_{t_2} = \dot{m}(h_4 - h_5)$$

From the steam tables,

$$h_4 = 3{,}069.3 \text{ kJ/kg}$$
$$s_4 = 7.7022 \text{ kJ/kg} - \text{K}$$

To find h_5, note that

$$s_4 = s_5$$

At state 5,

$$s_{f_5} = 0.6493 \text{ kJ/kg} - \text{K}$$
$$h_{f_5} = 191.83 \text{ kJ/kg}$$
$$s_{fg_5} = 7.5009 \text{ kJ/kg} - \text{K}$$
$$h_{fg_5} = 2{,}392.8 \text{ kJ/kg}$$
$$s_{g_5} = 8.1502 \text{ kJ/kg} - \text{K}$$
$$h_{g_5} = 2{,}584.7 \text{ kJ/kg}$$
$$s_4 = s_5 = s_{f_5} + x_5 s_{fg_5}$$
$$x_5 = 0.949$$
$$h_5 = h_{f_5} + x_5 h_{fg_5}$$
$$h_5 = 191.83 + 0.949(2{,}392.8)$$
$$h_5 = 2{,}463 \text{ kJ/kg}$$

$$\therefore \frac{\dot{W}_{t_2}}{\dot{m}} = h_4 - h_5$$

$$= 3{,}069.3 - 2{,}463$$
$$= 606 \text{ kJ/kg}$$

To obtain $\dot{W}_p$, assume that $\dot{W}_p = \dot{m}\, v_6(p_1 - p_6)$.

From the steam tables,

$$v_6 = v_{f_6}$$
$$= 1.0102 \times 10^{-3} \text{ m}^3\text{/kg}$$

Thus,

$$\frac{\dot{W}_p}{\dot{m}} = 1.0102(30 - 0.1)10^5 \times 10^{-6}$$
$$= 3.0 \text{ kj/kg}$$

To obtain $\dot{Q}_b$, use

$$\dot{Q}_b = \dot{m}(h_2 - h_1)$$

$$h_1 = h_6 + \frac{\dot{W}_p}{\dot{m}}$$
$$= 191.8 + 3.0$$
$$= 194.8 \text{ kJ/kg}$$

$$\frac{\dot{Q}_b}{\dot{m}} = 2,993.5 - 194.8$$
$$= 2,799 \text{ kJ/kg}$$

To find $\dot{Q}_r$,

$$\dot{Q}_r = \dot{m}(h_4 - h_3)$$

$$\frac{\dot{Q}_r}{\dot{m}} = 3,069.3 - 2,542$$
$$= 527 \text{ kJ/kg}$$

From equation (1) then

$$\eta = \frac{452 + 606 - 3}{2,799 + 527}$$
$$= 0.317$$

PROBLEM 3

A container which has a volume of 0.1m³ is fitted with a plunger enclosing 0.5 kg of steam at 0.4 MPa. Calculate the amount of heat transferred and the work done when the steam is heated to 300°C at constant pressure.

SOLUTION

For this system changes in kinetic and potential energy are not significant. Therefore,

$$Q = m(u_2 - u_1) + W$$

$$W = \int_1^2 P dV = P\int_1^2 dV = P(V_2 - V_1) = m(P_2 v_2 - P_1 v_1)$$

Therefore,

$$Q = m(u_2 - u_1) + m(P_2 v_2 - P_1 v_1) = m(h_2 - h_1)$$

$$v_1 = \frac{V_1}{m} = \frac{0.1}{0.5} = 0.2 = 0.001084 + x_1(0.4614)$$

$$x_1 = \frac{0.1989}{0.4614} = 0.4311$$

Then

$$\begin{aligned} h_1 &= h_f + x_1 h_{fg} \\ &= 604.74 + 0.4311 \times 2133.8 = 1{,}524.6 \\ h_2 &= 3{,}066.8 \\ Q &= 0.5(3{,}066.8 - 1{,}524.6) = 771.1 \text{ kJ} \\ W &= mP(v_2 - v_1) = 0.5 \times 400(0.6548 - 0.2) \\ &= 91.0 \text{ kJ} \end{aligned}$$

Therefore,

$$U_2 - U_1 = Q - W = 771.1 - 91.0 = 680.1 \text{ kJ}.$$

The heat transfer can be calculated from u_1 and u_2 by using

$$\begin{aligned} Q &= m(u_2 - u_1) + W \\ u_1 &= u_f + x_1 u_{fg} \\ &= 604.31 + 0.4311 \times 1{,}949.3 = 1{,}444.6 \\ u_2 &= 2{,}804.8 \end{aligned}$$

PROBLEM 5

Steam leaves the boiler in a steam turbine plant at 2 MPa, 300°C and is expanded to 3.5 kPa before entering the condenser. Compare the following four cycles:

(1) A superheated Rankine cycle.

(2) A reheat cycle, with steam reheated to 300°C at the pressure when it becomes saturated vapor.

(3) A regenerative cycle, with an open feedwater heater operating at the pressure where steam becomes saturated vapor.

(4) A regenerative cycle, with a closed feedwater heater operating at the pressure where steam becomes saturated vapor.

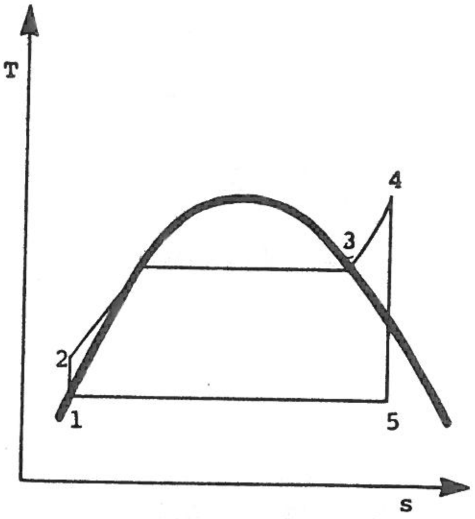


Figure 38. Rankine cycle

SOLUTION

(1) Referring to Figure 38, the steam tables show that

$$h_4 = 3{,}025 \text{ kJ/kg}$$
$$s_4 = 6.768 \text{ kJ/kg} - \text{K}$$

At $P = 3.5$ kPa,

$$s_g = 8.521 \text{ kJ/kg} - \text{K}$$
$$s_f = 0.391 \text{ kJ/kg} - \text{K}$$

Since $s_5 = s_4$, steam at 5 is a mixture of liquid and vapor. The quality is found as

$$\begin{aligned} x_5 &= \frac{s_5 - s_f}{s_{fg}} \\ &= \frac{6.768 - 0.391}{8.130} \\ &= 0.785 \end{aligned}$$

Therefore,

$$\begin{aligned} h_5 &= h_f + x_5 h_{fg} \\ &= 112 + 0.785(2{,}438) \\ &= 2{,}023 \text{ kJ/kg} \end{aligned}$$

hence

$$\begin{aligned} w_{45} &= h_4 - h_5 \\ &= 3{,}025 - 2{,}023 \\ &= 1{,}002 \text{ kJ/kg} \end{aligned}$$

Now

$$\begin{aligned} w_{12} &= h_1 - h_2 \\ &= v_f(p_1 - p_2) \\ &= 0.0010(0.0035 - 2) \times 10^3 \text{ kJ/kg} \\ &= -2 \text{ kJ/kg} \end{aligned}$$

Therefore, the net work output is

$$w = w_{45} + w_{12} = 1{,}000 \text{ kJ/kg}$$

Heat input is

$$q_{42} = h_4 - h_2$$

But

$$h_2 = h_1 - w_{12} = 112 + 2 = 114 \text{ kJ/kg}$$

therefore,

$$q_{42} = 3{,}025 - 114 = 2{,}911 \text{ kJ/kg}$$

Thus,

$$\eta = \frac{w}{q_{42}} = \frac{1{,}000}{2{,}911} = 0.344$$

Also

$$\text{Specific Steam Consumption} = \frac{3,600}{w} = \frac{3,600}{1,000} = 3.6 \text{ kg/kWh}$$

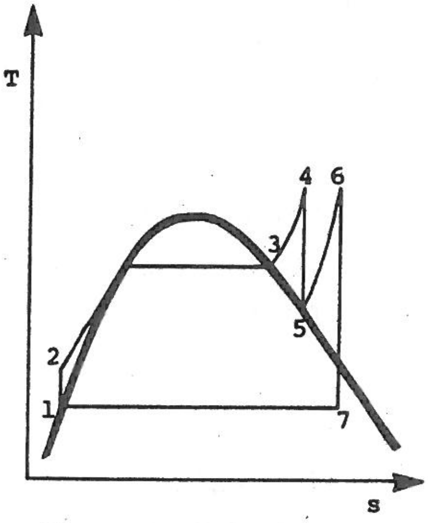


Figure 39. Reheat cycle

(2) Refer to Figure 39, and note that since

$$s_5 = s_{sat} = s_4 = 6.768 \text{ kJ/kg} - \text{K}$$

the pressure at reheat point 5 can be found using the steam tables. Interpolating between 0.55 MPa and 0.6 MPa gives

$$P_5 = 0.588 \text{ MPa}.$$

Then

$$\begin{aligned} h_5 &= 2,753 + \frac{0.588 - 0.55}{0.60 - 0.55}(2,757 - 2,753) \\ &= 2,753 + \frac{0.038}{0.05} \times 4 \\ &= 2,756 \text{ kJ/kg} \end{aligned}$$

As 6 and 5 are on the same isobar, by interpolation

$$\begin{aligned} h_6 &= 3,065 + \frac{0.588 - 0.5}{0.60 - 0.5}(3,062 - 3,065) \\ &= 3,065 + \frac{0.088}{0.1}(-3) \\ &= 3,062.4 \text{ kJ/kg} \\ s_6 &= 7.460 + 0.88(7.373 - 7.460) \\ &= 7.460 + 0.88(-0.087) \\ &= 7.384 \text{ kJ/kg} - \text{K} \end{aligned}$$

At $P = 3.5$ kPa,

$$\begin{aligned} s_g &= 8.521 \text{ kJ/kg} - \text{K} \\ s_f &= 0.391 \text{ kJ/kg} - \text{K} \end{aligned}$$

Since $s_7 = s_6$, the quality at 7 is found as

$$x_7 = \frac{7.384 - 0.391}{8.130} = 0.86.$$

Then

$$\begin{aligned} h_7 &= 112 + 0.86(2,438) \\ &= 112 + 2,095 = 2,207 \text{ kJ/kg} \end{aligned}$$

The net work output is given by

$$\begin{aligned} w &= w_{45} + w_{67} + w_{12} \\ &= (3,025 - 2,765) + (3,062.4 - 2,207) - 2 \\ &= 1,122.4 \end{aligned}$$

The heat input is

$$\begin{aligned} q &= q_{42} + q_{65} \\ &= 2,911 + (h_6 - h_5) \\ &= 2,911 + (3,062.4 - 2,756) \\ &= 3,217.4 \end{aligned}$$

Therefore,

$$\eta = \frac{1,122.4}{3,217.4} = 0.349$$

and

$$\text{s.s.c.} = \frac{3{,}600}{w} = \frac{3{,}600}{1{,}122.4} = 3.2 \text{ kg/kWh.}$$

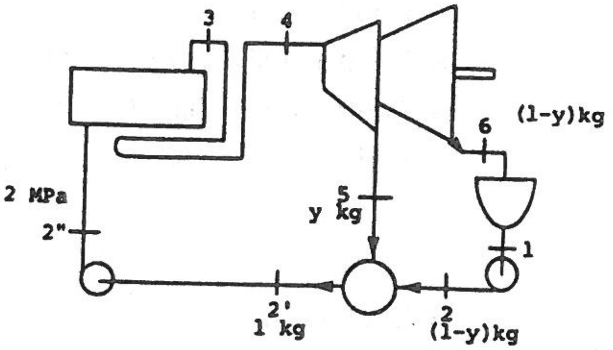


Figure 40. (a) Equipment schematic for regenerative cycle

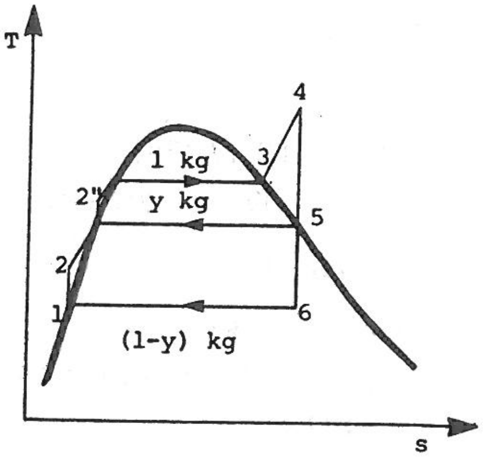


Figure 40. (b) Regenerative cycle

(3) Refer to Figures 40 (a) and 40 (b). The work is as in (b)

$$w_{45} = 269 \text{ kJ/kg}$$

Next determine the amount of steam bled off at 5. Consider an energy balance for the open feedwater heater with

$$h_{2'} = yh_s - (1-y)h_2$$

which gives

$$y = \frac{h_{2'} - h_2}{h_5 - h_2}$$

To find the value for $h_{2'}$, enter the steam tables. At 5 the pressure is known ($P = 0.588$ MPa) and the state of the steam is given as saturated vapor. Therefore, by interpolating between the values of 0.5 MPa and 0.6 MPa, obtain

$$\begin{aligned} h_{2'} &= 656 + \frac{0.588 - 0.55}{0.60 - 0.55}(670 - 656) \\ &= 656 + \frac{0.038}{0.05} \times 14 \\ &= 666.6 \text{ kJ/kg} \end{aligned}$$

Then

$$\begin{aligned} y &= \frac{666.6 - 114}{2,756 - 114} \\ &= \frac{552.6}{2,642} \\ &= 0.209 \end{aligned}$$

Hence,

$$\begin{aligned} w_{56} &= (1 - y)(h_5 - h_6) \\ &= 0.791(2,756 - 2,023) \\ &= 580 \text{ kJ/kg} \end{aligned}$$

also

$$\begin{aligned} w_{2'2''} &= v_f(P_{2'} - P_{2''}) \\ &= 0.0011(0.588 - 2) \times 10^3 \\ &= -1.1 \times 1.412 \\ &= -1.55 \text{ kJ/kg} \end{aligned}$$

Therefore,

$$\begin{aligned} w &= w_{45} + w_{56} + w_{12} + w_{2'2''} \\ &= 269 + 580 - 0.791 \times 2 - 1.55 \\ &= 845.87 \text{ kJ/kg} \end{aligned}$$

The heat input is

$$q_{42''} = 3{,}025 - (666.6 + 1.55)$$
$$= 2{,}356.8 \text{ kJ/kg}$$

The efficiency of this cycle is

$$\eta = \frac{w}{q_{42''}} = \frac{845.87}{2{,}356.8} = 0.3595$$

and

$$\text{s.s.c.} = \frac{3{,}600}{w} = \frac{3{,}600}{845.9} = 4.25 \text{ kg/k Wh..}$$

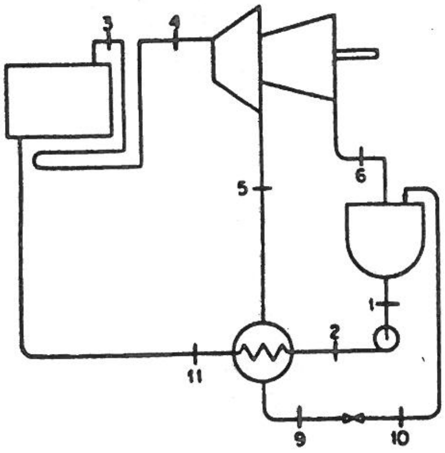


Figure 41. (a) Equipment diagram including closed heater

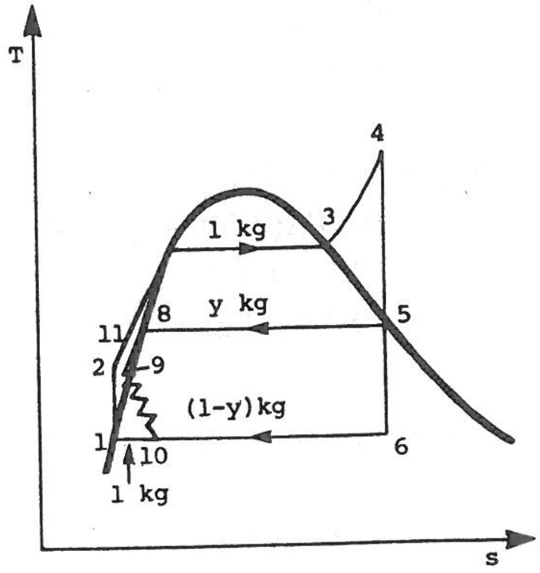


Figure 41. (b) A regenerative cycle with closed heater

(4) Refer to Figures 41 (a) and 41 (b). The work is as in part (b).

$$w_{45} = 269 \text{ kJ/kg}$$

Heat balance for the heater as a closed system gives

$$h_{21} = yh_5 - (1-y)h_2$$

giving

$$y = \frac{h_{11} - h_2}{h_5 - h_9}$$

Now in finding the enthalpies in the feed line, it is usual to make the following assumptions:

i. Neglect the feed pump term.

ii. Assume the enthalpy of the compressed liquid to be the same as that of the saturated liquid at the same temperature.

iii. Assume the states of the condensate extracted from the turbine, before and after throttling, to be the same as that of the saturated liquid at the lower pressure of the throttled liquid.

Using these assumptions

$$h_2 = h_1$$
$$h_{11} = h_8$$
$$h_9 = h_{10} = h_1$$

whence

$$y = \frac{h_8 - h_1}{h_5 - h_1}$$
$$= \frac{666.6 - 112}{2,756 - 112} = 0.209 \text{ kJ/kg}$$

Also,

$$w_{56} = 580 \text{ kJ/kg.}$$

Therefore,

$$w = w_{45} + w_{56} + w_{12}$$
$$= 269 + 580 - 2 = 847 \text{ kJ/kg}$$

Heat input $q_{411} = 2{,}358.4$ kJ/kg.

Then

$$\eta = \frac{w}{q_{411}} = \frac{847}{2{,}358.4} = 0.360$$

and

$$\text{s.s.c.} = \frac{3{,}600}{w} = \frac{3{,}600}{847} = 4.25 \text{ kg/k Wh.}$$

PROBLEM 6

(1) One kilogram of air at 101.35 kPa, 21°C is compressed in an Otto cycle with a compression ratio of 7 to 1. During the combustion process, 953.66 kJ of heat is added to the air. Compute (a) the specific volume, pressure, and temperature at the four points in the cycle, (b) the air standard efficiency, and (c) the mep (mean effective pressure) and hp of the engine, if it uses 1 kg/min of air.

(2) Calculate the efficiency for a Carnot cycle operating between the maximum and minimum temperatures of the Otto cycle (Figure 42).

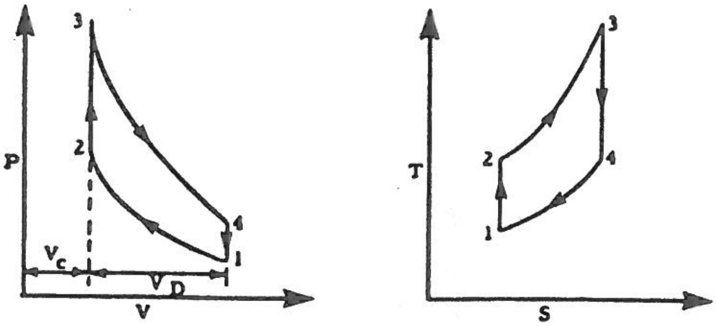


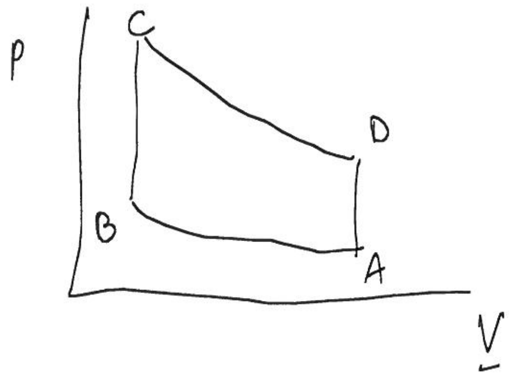
Figure 42. Otto cycle

SOLUTION

(1) (a) At state 1,

$$P_1 = 101.35 \text{ kPa}$$
$$T_1 = 294\text{K}$$

Fall 2007 Exam #2, Problem #3



$V_A/V_B = 8$

$P_A = 100 \text{kPa}$

$Q_{BC} = +800 \text{ KJ/kg}$

$C_p = 7/2 R$

AB : reversible, adiabatic

BC : isochoric

CD : reversible, adiabatic

D,A : isochoric

	T (K)	P (kPa)
A	290	100
B	662	1838
C	1782	4918
D	776	268

Adiabatic compression

$$\cancelto{0}{\frac{dS}{dt}} = \sum \dot{m}_k \hat{S}_k + \cancelto{0}{\frac{\dot{Q}}{T}} + \cancelto{0}{\dot{S}_{gen}}$$

$$\hat{S}_A = \hat{S}_B$$

$$\therefore \quad \Delta \hat{S}(P, \underline{V}) = 0 = C_p \ln \frac{\underline{V}_B}{\underline{V}_A} + C_v \ln \frac{P_B}{P_A}$$

$$P_B / P_A = \left(\frac{\underline{V}_A}{\underline{V}_B} \right)^{C_p / C_v}$$

$$P_B = P_A (8)^{\frac{7/\cancel{2}\cancel{R}}{5/\cancel{2}\cancel{R}}}$$

$$= 100 (8)^{7/5}$$

Also,

$$\Delta \hat{S}(T, \underline{V}) = 0 \Longrightarrow T_B / T_A = \left(\frac{\underline{V}_A}{\underline{V}_B} \right)^{\left(\frac{C_p - C_v}{C_v} \right)}$$

$$T_B = (290\text{K}) (8)^{(7/5 - 1)}$$

$$T_B = 666.2 \text{ K}$$

Balance from B → C

$$\frac{dU}{dt} = \cancel{\sum_k \dot{m}_k \hat{H}_k}^{\,0} + \dot{Q} + \cancel{\dot{W}}^{\,0}$$

$$\dot{m}\left(\hat{U}_C - \hat{U}_B\right) = \dot{Q}$$

$$\frac{\dot{Q}}{\dot{m}} = \hat{U}_C - \hat{U}_B = c_V\left(T_C - T_B\right)$$

$$T_C = \frac{\dot{Q}}{\dot{m}\,c_V} + T_B$$

$$= \frac{800\ \cancel{kJ}/\cancel{kg}\quad 29\ \cancel{g}/\cancel{mol}\cdot\frac{\cancel{kg}}{1000\cancel{g}}\cdot\frac{1000\cancel{J}}{\cancel{kJ}}}{(5/2)(8.314\ \cancel{J}/\cancel{mol}\cdot K)} + 666.2\ K$$

$$T_C = 1782.4\ K$$

$$\frac{P_C \underline{V}_C}{T_C} = \frac{P_B \underline{V}_B}{T_B}$$

$$P_C = P_B\left(\frac{T_C}{T_B}\right)\cancel{\left(\frac{\underline{V}_B}{\underline{V}_C}\right)}^{\,1\ (\text{isochoric})}$$

$$P_C = (1838\,\text{kPa})\left(\frac{1782.4}{666.2}\right)$$

$$P_C = 4917.5\ \text{kPa}$$

As before during adiatic compression

$$P_D/P_C = \left(\frac{\underline{V}_C}{\underline{V}_D}\right)^{C_P/C_V}$$

$$\text{\&}\quad T_D/T_C = \left(\frac{\underline{V}_C}{\underline{V}_D}\right)^{\left(\frac{C_P - C_V}{C_V}\right)}$$

$$\underline{V}_D = \underline{V}_A$$

$$\underline{V}_B = \underline{V}_C$$

$$P_D = P_C\left(\frac{\underline{V}_B}{\underline{V}_A}\right)^{C_P/C_V}$$

$$= 4917.5\,\text{kPa}\left(\frac{1}{8}\right)^{7/5}$$

$$P_D = 267.6\ \text{kPa}$$

$$T_D = T_C \left(\frac{\underline{V}_B}{\underline{V}_A}\right)\left(\frac{c_p - c_v}{c_v}\right)$$

$$= 1782.4 \left(\frac{1}{8}\right)^{(7/5 - 1)}$$

$$T_D = 775.8K$$

Overall Balance

$$\cancelto{0}{\frac{dU}{dt}} = \cancelto{0}{\sum \dot{m}_k \hat{H}_k} + Q + \dot{W}_{net}$$

$$-\dot{W}_{Net} = Q_{in} + Q_{out}$$

Need Q_{out}

Balance around last isochoric step

$$\frac{dU}{dt} = \cancelto{0}{\sum \dot{m}_k \hat{H}_k} + \dot{Q}_{out} + \cancelto{0}{\dot{W}}$$

$$\dot{m}\left(\hat{U}_A - \hat{U}_D\right) = \dot{Q}_{out}$$

$$\frac{\dot{Q}_{out}}{\dot{m}} = c_v\left(T_A - T_D\right)$$

$$\frac{\dot{Q}_{out}}{\dot{m}} = \frac{\frac{5}{2}\left(8.314\,\cancel{J}/\cancel{mol}\cdot K\right)}{29\ \cancel{g}/\cancel{mol}} \cdot \frac{\cancel{1000g}}{Kg} \cdot \frac{KJ}{\cancel{1000J}}\left(290 - 775.8\right)$$

$$\frac{\dot{Q}_{out}}{\dot{m}} = -348.2\ KJ/Kg$$

$$-W_{net} = 800 - 348.2$$

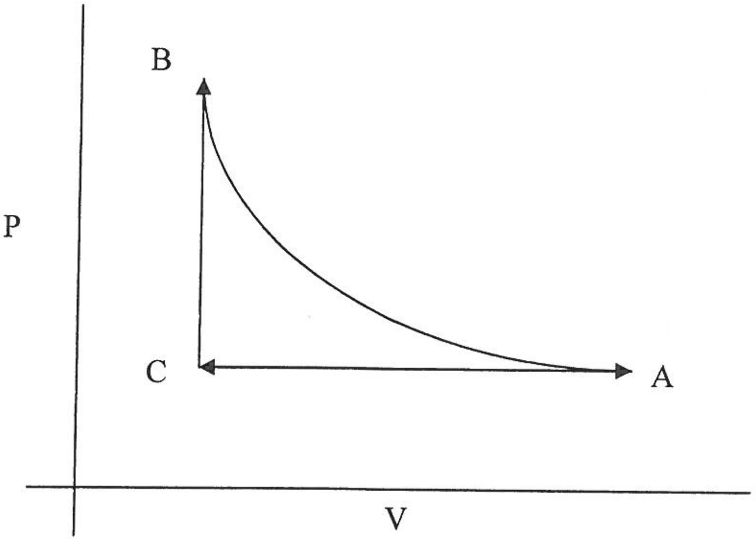
$$-W_{net} = 451.8\ KJ/Kg$$

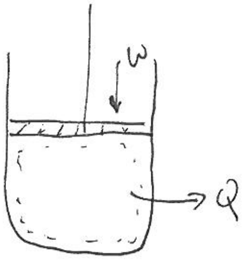
$$\frac{-W_{net}}{Q_{in}} = \frac{451.8}{800} \Rightarrow \eta = 0.565$$

Process Thermodynamics

First Law
Example 1

A gas is confined in a cylinder by a piston. It is taken from state A to state B along the path ACB as shown on the PV diagram below. The process from A to C is constant pressure, and the system receives 50 J of work and gives up 25 J of heat to the surroundings. The process from C to B is constant volume and the system receives 75 J of heat. The return path from B to A is adiabatic. How much work is exchanged with the surroundings for the adiabatic path? Assume all processes are ~~reversible~~ frictionless.





System: gas in cylinder

Known:

$Q_{AC} = -25\text{ J}$

$W_{AC} = 50\text{ J}$

$Q_{CB} = 75\text{ J}$

$Q_{BA} = 0$

$\Delta U_{BA} = -\Delta U_{AB}$

$$\Delta U + \cancelto{0}{\Delta E_p} + \cancelto{0}{\Delta E_k} = Q + W$$

$$\Delta U = Q + W$$

Assume: reversible (details to follow later in course) (i.e. frictionless)

$$\Delta U_{BA} = \cancelto{0}{Q_{BA}} + W_{BA}$$

$$\Delta U = 0 = \Delta U_{AC} + \Delta U_{CB} + \Delta U_{BA}$$

$$-\Delta U_{AB} = \Delta U_{BA} = -\Delta U_{AC} - \Delta U_{CB}$$

$\hookrightarrow$ U is a state function – we know that we can get same ΔU by following different path

$$\Delta U_{AC} = q_{AC} + W_{AC}$$

$$= -25\,J + 50\,J$$

$$= 25\,J$$

$$\Delta U_{CB} = q_{CB} + \cancel{W_{CB}}^{\,0}$$

$$= 75\,J$$

$$W_{CB} = \int_{V_C}^{V_B} p\,dV$$

$$= 0$$

$$\Delta U_{BA} = -\Delta U_{AC} - \Delta U_{CB}$$

$$= -25J - 75\,J$$

$$= -100\,J$$

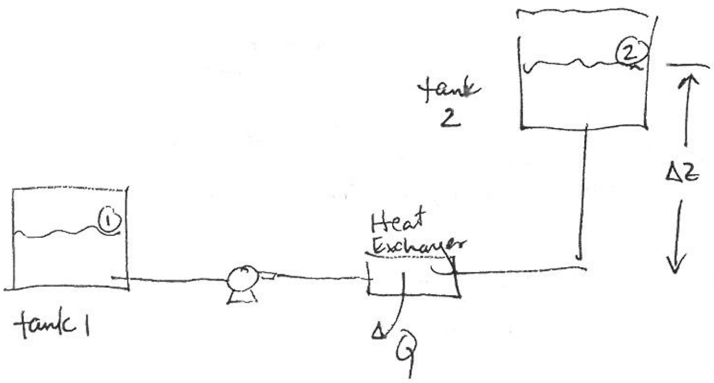
$$\Delta U_{BA} = W_{BA}$$

$$\boxed{W_{BA} = -100\,J}$$ work done by system

Process Thermodynamics

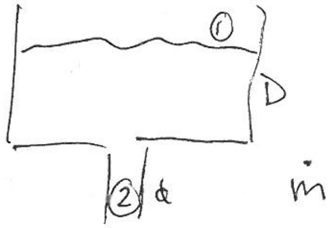
First Law
Example 2

Water at 200 °F is pumped from a storage tank at the rate of 50 gal min^{-1}. The motor for the pump supplies work at the rate of 2 hp. The water goes through a heat exchanger, giving up heat at the rate of 40 000 Btu min^{-1}, and is delivered to a second storage tank at an elevation 50 ft above the first tank. What is the temperature of the water delivered to the second tank? The density of water at 200 °F is 60.1 lb$_m$ ft^{-3}.



Known:
$T_1 = 200°F$
$\dot{V}_1 = 50$ gal/min
$W_{pump} = 2$ hp
$\dot{Q} = -40\,000$ Btu/min
$\Delta Z = 50$ ft
$\rho = 60.1$ lbm/ft^3

Assume
- No friction
- P effects in tank negligible
  - Height of water in tanks not important
- (want) $u_1 \sim 0$, $u_2 \sim 0$] – tanks are large (do derivation if they don't know)
- (want) $g = 32.174$ ft/s^2
- (want) close to sat liquid

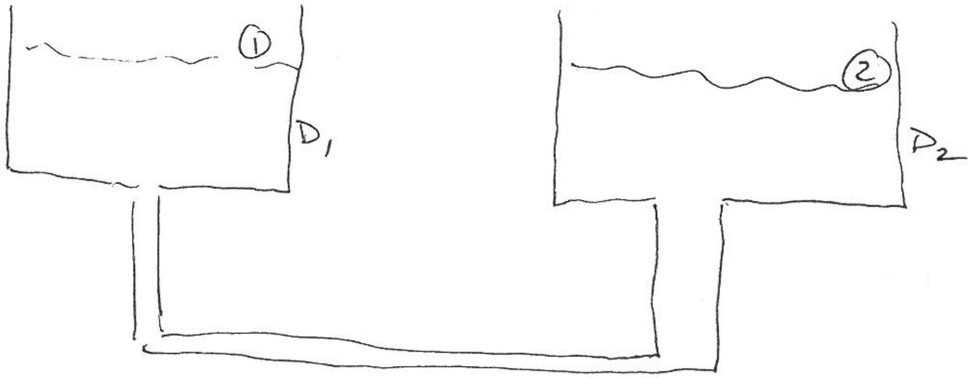


D>>d

$$\dot{m}_1 = \dot{m}_2$$
$$\rho_1 u_1 A_1 = \rho_2 u_2 A_2$$
$$\rho_1 = \rho_2$$
$$u_1 A_1 = u_2 A_2$$
$$u_1 = u_2 \frac{\cancel{\pi/4}\, d^2}{\cancel{\pi/4}\, D^2}$$
$$u_1 = u_2 \left(\frac{d}{D}\right)^2$$

since D>>d

$u_1 \sim 0$



$$u_1 D_1^2 = u_2 D_2^2$$

if $u_1 \sim 0$, then $u_2 \sim 0$

Energy Balance

(derivation if necessary)

$$\Delta\hat{H} + \cancelto{0}{\frac{u_2^2 - u_1^2}{2}} + g(z_2 - z_1) = Q + W_s$$

$$\Delta\hat{H} = \hat{H}_2 - \hat{H}_1 = Q + W_s - g\Delta z$$

$$\hat{H}_2 = \hat{H}_1 + Q + W_s - g\Delta z$$

Use steam tables to get $\hat{H}_1$

Enthalpy of liquid H_2O @ 200°F — close to sat liq — more later in Semester...

$\hat{H}_1 = 168.09$ Btu/lbm

$$\hat{H}_2 = (168.09\ \text{Btu/lbm}) + (-40\,000\ \text{Btu/min})\overbrace{\left(\frac{\text{min}}{50\ \text{gal}}\right)\left(\frac{\text{ft}^3}{60.11\ \text{lbm}}\right)\left(\frac{7.48\ \text{gal}}{\text{ft}^3}\right)}^{\text{mass flow rate}}$$

$$+ (2\ \text{hp})\left(\frac{42.41\ \text{Btu/min}}{1\ \text{hp}}\right)\left(\frac{1}{50}\right)\left(\frac{7.48}{60.1}\right)\left(\frac{\text{min}}{\text{lbm}}\right)$$

assume g

$$- \frac{(32.174\ \text{ft/s}^2)(50\ \text{ft})}{32.174\ \frac{\text{lbm}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}} \cdot \frac{\text{Btu}}{778.16\ \text{ft}\cdot\text{lbf}}$$

$$\hat{H}_2 = 68.74\ \text{Btu/lbm}$$

From steam tables, we see that T_2 is between 100 & 102°F

Interpolate to get $\boxed{T_2 = 100.74\,°\text{F}}$

Note that W_s & $\Delta E_p \sim 0$ in comparison to Q & could have been neglected more constants — Air (low range)

closer to range

a 28.088

b. 0.197×10^{-2}

c. 0.480×10^{-5}

d -1.965×10^{-9}

C_p — J/mol·K

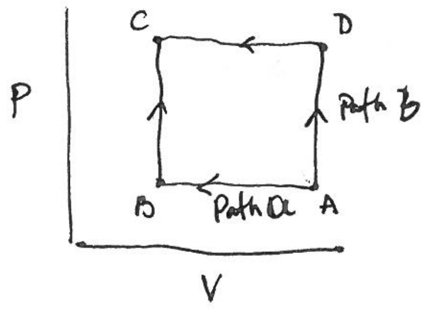
Process Thermodynamics

First Law
Example 3

Air at 1 bar and 298.15 K is compressed to 5 bar and 298.15 K by two different frictionless (reversible) processes:

(a) Cooling at constant pressure followed by heating at constant volume
(b) Heating at constant volume followed by cooling at constant pressure

Calculate the heat and work requirements and the change in internal energy and enthalpy of the air for each path. Assume that air is an ideal gas, regardless of the changes it undergoes and that $C_p = C_v + R$. At 298.15 K and 1 bar the molar volume of air is 0.02479 $m^3\ mol^{-1}$.



Known: Air

$P_A = 1$ bar

$P_C = 5$ bar

$T_A = T_C = 298.15$ K

$\underline{V}(T_A, P_A) = 0.02479\ m^3/mol$

$W_s = 0$ (wait)

Assume: 1 mol (wait)

$\Delta E_p = \Delta E_K = 0$

(a) First Step (A → B) closed system changing V
Const. P process

$$\Delta H + \cancel{\Delta E_p}^{0} + \cancel{\Delta E_K}^{0} = Q + \cancel{W_s}^{0}$$ (add to known)

$$\Delta H = Q$$

$$\Delta H_{AB} = Q_{AB}$$

$$\boxed{\Delta H_{AB} = Q_{AB} = \int_{T_A}^{T_B} c_p\, dT}$$; need c_p & T_B

$$= \int_{T_A}^{T_B} a + bT + cT^2 + dT^3$$

$$= a(T_B - T_A) + \frac{b}{2}(T_B^2 - T_A^2) + \frac{c}{3}(T_B^3 - T_A^3) + \frac{d}{4}(T_B^4 - T_A^4)$$

$$P\underline{V} = RT$$

$$\frac{P_i \underline{V}_i}{T_i} = \text{constant}$$

$$\frac{\cancel{P_A} \underline{V}_A}{T_A} = \frac{\cancel{P_B} \underline{V}_B}{T_B} \quad ; \; P_A = P_B$$

$$T_B = \frac{\underline{V}_B}{\underline{V}_A} T_A \quad \text{— Need} \qquad \text{but } \underline{V}_B = \underline{V}_C$$

$$T_B = \frac{\underline{V}_C}{\underline{V}_A} T_A \quad \text{— Need}$$

We also know

$$\frac{P_A \underline{V}_A}{T_A} = \frac{P_C \underline{V}_C}{T_C} \quad ; \; T_A = T_C$$

$$\underline{V}_C = \frac{P_A}{P_C} \underline{V}_A$$

sub

$$T_B = \frac{P_A}{P_C} \cancel{\underline{V}_A} \, T_A$$

$$T_B = \left(\frac{1\text{ bar}}{5\text{ bar}}\right)(298.15\text{ K})$$

$$= 59.63\text{ K}$$

For ΔU,

$$H = U + P\underline{V}$$

$$\Delta H = \Delta U + \Delta(P\underline{V})$$

$$\Delta U_{AB} = \Delta H_{AB} - \Delta(P\underline{V})_{AB}$$

$$= \Delta H_{AB} - P_A(\underline{V}_B - \underline{V}_A) \quad ; \; \underline{V}_B = \underline{V}_C = \frac{P_A}{P_C}\underline{V}_A \text{ (above)}$$

$$= \Delta H_{AB} - P_A\left(\frac{P_A}{P_C}\underline{V}_A - \underline{V}_A\right)$$

$= \Delta H_{AB} - P_A \underline{V}_A \, (P_A$

Second step — closed system at const. V (B->C)

$$\Delta U + \cancelto{0}{\Delta E_p} + \cancelto{0}{\Delta E_k} = Q + \cancelto{0}{W} \text{ (const. V)}$$

$$\Delta U_{BC} = Q_{BC} = \int_{T_B}^{T_C} C_v\,dT \qquad ; \; C_v = C_p - R$$

$$= \int_{T_B}^{T_B} C_p - R = (a-R)(T_B - T_B) + \frac{b}{2}(T_B^2 - T_B^2) + \frac{c}{3}(T_C^3 - T_B^3) + \frac{d}{4}(T_C^4 - T_B^4)$$

$$\boxed{\text{Entire process}}$$

$$Q_T = Q_{AB} + Q_{BC}$$

$$\Delta U_T = \Delta U_{AB} + \Delta U_{BC}$$

★ extensive properties are additive

$$\Delta H_T = \Delta H_{AB} + \Delta H_{BC}$$

$$= \Delta U + \cancelto{0}{\Delta(PV)}$$

~~where~~ because $\frac{P_A \underline{V}_A}{T_A} = \frac{P_C \underline{V}_C}{T_C}$

or $P_A \underline{V}_A = P_C \underline{V}_C$

Overall Balance

$$\cancelto{0}{\Delta U} = Q + W$$

$$W = -Q$$

Final Values

$\Delta H_{AB} = Q_{AB} = -6820$ J

$\Delta U_{AB} = -4837$ J

$\Delta U_{BC} = Q_{BC} = 4837$ J

$Q_T = -1983$ J

$\Delta U_T = 0$

$\Delta H_T = 0$

$W = 1983$ J

Recall that $\Delta U = f(T)$ for IG

(2) In similar fashion, for path (b)

A → D

$$\frac{P_A \underline{V}_A}{T_A} = \frac{P_D \underline{V}_D}{T_D} \;; \quad \underline{V}_A = \underline{V}_D \;; \quad P_D = P_C$$

$$T_D = \frac{P_C}{P_A} T_A$$

$$= 1490.75 \text{ K}$$

$$Q_{AD} = \Delta U_{AD} = \int_{T_A}^{T_D} C_v \, dT$$

$$= (a-R)(T_D - T_A) + \frac{b}{2}(T_D^2 - T_A^2) + \frac{c}{3}(T_D^3 - T_A^3)$$
$$+ \frac{d}{4}(T_D^4 - T_A^4)$$

D → C

$$Q_{DC} = \Delta H_{DC} = \int_{T_D}^{T_C} C_p \, dT$$

$$= a(T_C - T_D) + \frac{b}{2}(T_C^2 - T_D^2) + \frac{c}{3}(T_C^2 - T_D^2)$$
$$+ \frac{d}{4}(T_C^4 - T_D^4)$$

$$\Delta U_{DC} = \Delta H_{DC} - \Delta (P\underline{V})_{DC}$$

$$= \Delta H_{DC} - P_C(\underline{V}_C - \underline{V}_D)$$

$$\underline{V}_D = \underline{V}_A$$
$$T_A = T_C$$

$$\frac{P_A \underline{V}_A}{\cancel{T_A}} = \frac{P_C \underline{V}_C}{\cancel{T_C}}$$

$$\underline{V}_C = \frac{P_A}{P_C} \underline{V}_A$$

$$= \Delta H_{DC} - P_C\left(\frac{P_A}{P_C} \underline{V}_A - \underline{V}_A\right)$$

$$= \Delta H_{DC} - P_C \underline{V}_A \left(\frac{P_A}{P_C} - 1\right)$$

Entire Process

$$Q_T = Q_{AD} + Q_{DC}$$

$$\Delta U_T = \Delta U_{AD} + \Delta U_{DC}$$

$$\Delta H_T = \Delta H_{AD} + \Delta H_{DC}$$

Overall Balance

$$\Delta U = Q + W$$

$$W = -Q$$

Final Values

$\Delta H_{DC} = Q_{DC} = -38{,}435\ \text{J}$

$\Delta U_{DC} =$ [redacted] $-28{,}520\ \text{J}$

$Q_{AD} = \Delta U_{AD} = 28{,}520\ \text{J}$

$Q_T = -9915\ \text{J}$

$\Delta U_T = \Delta H_T = 0$

$W = 9915\ \text{J}$

☆ Note that the property changes ΔU & ΔH are the same for both paths, but Q & W are path-dependent

Process Thermodynamics

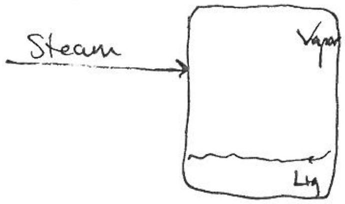
Need to clarify that the Final & Steam conditions can be known for P alone

First Law
Example 4

A well-insulated storage tank of 60 m3 contains 200 L of liquid water at 75 °C. The rest of the tank contains steam in equilibrium with the water. Spent process steam at 2 bar and 90% quality enters the storage tank until the pressure in the tank reaches 2 bar. ~~over [illegible] takes~~ Assuming that the heat losses from the system to the tank and the environment are negligible, calculate the total amount of steam that enters the tank during the filling process and the fraction of liquid water present at the end of the process.

$D_{tank} =$

$D_{pipe} =$



$Q = 0$

$W = 0$

Initial	Final	Stream in
$V = 60\ m^3$	$P^f = 2$ bar	$P = 2$ bar
$V_L^i = 200L$		$X_V = 0.9$
$T^i = 75°C$		
Sat.		

can use steam tables $\hat{V}, \hat{u}, \hat{H}$
↳ info on

Assume $\Delta E_p = \Delta E_K \sim 0$

~~Have students try~~

$\hat{V}_L^i = 1.026 \times 10^{-3}\ m^3/kg$

$\hat{V}_V^i = 4.131\ m^3/kg$

$\hat{U}_L^i = 313.9\ kJ/kg$

$\hat{U}_V^i = 2475.9\ kJ/kg$

$\hat{V}_L^f = 1.061 \times 10^{-3}\ m^3/kg$

$\hat{V}_V^f = \cancel{4.131}\ 0.8857\ m^3/kg$

$\hat{u}_L^f = 504.49\ kJ/kg$

$\hat{u}_V^f = 2529.5\ kJ/kg$

Have students get these

$\hat{H}_{L,in} = 504.7\ kJ/kg$

$\hat{H}_{V,in} = 2706.7\ kJ/kg$

$$m^f - m^i = \boxed{\Delta m}$$ total mass of steam introduced

$$m_L^i + m_V^i = m^i$$

$$m_L^i = \frac{V_L^i}{\hat{V}_L^i} = \frac{200L}{1.026 \times 10^{-3}\ m^3/kg} \cdot \left(\frac{m^3}{1000L}\right) = 194.932\ kg$$

$$m_V^i = \frac{V_V^i}{\hat{V}_V^i} = \frac{60m^3 - 200L\left(\frac{m^3}{1000L}\right)}{4.131\ m^3/kg} = 14.476\ kg$$

$$m^i = 209.41\ kg$$

Energy Balance

w/o ΔE_p & ΔE_k

$$\frac{dU}{dt} = \dot{m}_{in}(\hat{H}_{in}) + \cancel{\dot{Q}}^{\,0} + \cancel{\dot{W}}^{\,0} \qquad (1)$$

integrate from initial to final conditions

$$\int_i^f dU = \int_i^f \dot{m}_{in}\hat{H}_{in}\,dt \qquad \dot{m}_{in}(t)$$

$$U^f - U^i = (m^f - m^i)\hat{H}_{in} \qquad (2)$$

~~$U^f = m^f\hat{U}^f + m^f$~~ $U^f = m^f\hat{U}^f$ (sub into (2))

$$\therefore\ m^f\hat{U}^f - m^i\hat{U}^i = (m^f - m^i)\hat{H}_{in} \qquad (3)$$

Use lever rule, for 2-phase mixture $\quad \Theta = x^{I}\Theta^{I} + x^{II}\Theta^{II}$

$$m\hat{U} = m_L\hat{U}_L + m_V\hat{U}_V \quad \text{(sub into (3))}$$

$$\left(m_L^f \overset{ST}{U_L^f} + m_V^f \overset{ST}{U_V^f}\right) - \left(\overset{\checkmark}{m_L^i}\,\overset{ST}{U_L^i} + \overset{\checkmark}{m_V^i}\,\overset{ST}{U_V^i}\right) = \left(m^f - \overset{\checkmark}{m^i}\right)\overset{ST}{\hat{H}_{in}} \qquad (4)$$

Analyze & Get variables $\qquad \hat{H}_{in} = 0.1\,\overset{\checkmark}{\hat{H}_{L,in}} + 0.9\,\overset{\checkmark}{\hat{H}_{V,in}}$

$$m^f = m_L^f + m_V^f$$

We have one eq & two variables — can we solve?
Do we know anything else about the final mass

$$\overset{\checkmark}{V} = m_L^f\,\overset{\checkmark}{\hat{V}_L^f} + m_V^f\,\overset{\checkmark}{\hat{V}_V^f} \qquad (5)$$

Now solve (4) & (5) simultaneously

$$504.49\,m_L^f + 2529.5\,m_V^f - 97030.3 = (m_L^f + m_V^f - 209.41)\,2486.5 \qquad (4)$$

$$9852.7 = 46.1\,m_L^f - m_V^f \qquad (4)$$

$$67.7 = 1.20\times10^{-3}\,m_L^f + m_V^f \qquad (5)$$

Note: [illegible]
Dn't needed

$$\boxed{\begin{array}{l} m_L^f = 215.19\ \text{kg} \\ m_V^f = 67.7\ \text{kg} \end{array}} \qquad \boxed{\Delta m = 73.48\ \text{kg}} \qquad \boxed{x_L = \frac{m_L^f}{m_L^f + m_V^f} = 0.761}$$

Process Thermodynamics

Engine Efficiency
Example 1

A central power plant, rated at 800,000 kW, generates steam at 585 K and discards heat to a river at 295 K. If the thermal efficiency of the plant is 70% of the maximum possible value, how much heat is discarded to the river at rated power?

Max efficiency given by $\eta_{max} = \frac{-W}{Q_H} = 1 - \frac{T_C}{T_H}$

$\eta_{max} = 1 - \frac{295}{585} = 0.496$ $\qquad \eta = 0.7\,\eta_{max} = 0.347$

Energy Bal

$$\left.\begin{aligned} Q_H + Q_C + W &= 0 \\ \eta &= \frac{-W}{Q_H} \end{aligned}\right\} \quad Q_C = +W\left(\frac{1}{\eta} - 1\right)$$

~~check seems~~ wrong!

$$= -800000\left(\frac{1}{0.347} - 1\right)$$

$$\boxed{Q_C = -1.505 \times 10^6 \text{ kW}}$$

Engine Efficiency
Example 2

The following heat engines produce power of 95,000 kW. Determine in each case the rates at which heat is absorbed from the hot reservoir and discarded to the cold reservoir.

1. A Carnot engine operates between heat reservoirs at 750 K and 300 K.
2. A practical engine operates between the same heat reservoirs but with a thermal efficiency η = 0.35.

1. $\eta = 1 - \frac{T_C}{T_H} = 1 - \frac{300}{750}$

$\eta = 0.6$

$\eta = \frac{-W}{Q_H}$

$Q_H = \frac{95000 \text{ kW}}{0.6}$

$$\boxed{Q_H = 1.583 \times 10^5 \text{ kW}}$$

$Q_C + Q_H + W = 0$

$Q_C + 1.583 \times 10^5 - 95000 = 0$

$$\boxed{Q_C = -6.33 \times 10^4 \text{ kW}}$$

2. $\eta = 0.35$

$$Q_H = \frac{-W}{\eta}$$

$$= \frac{95000}{0.35}$$

$$\boxed{Q_H = 2.71 \times 10^5 \text{ kW}}$$

$$Q_C + 2.71 \times 10^5 - 95000 = 0$$

$$\boxed{Q_C = -1.76 \times 10^5 \text{ kW}}$$

Process Thermodynamics

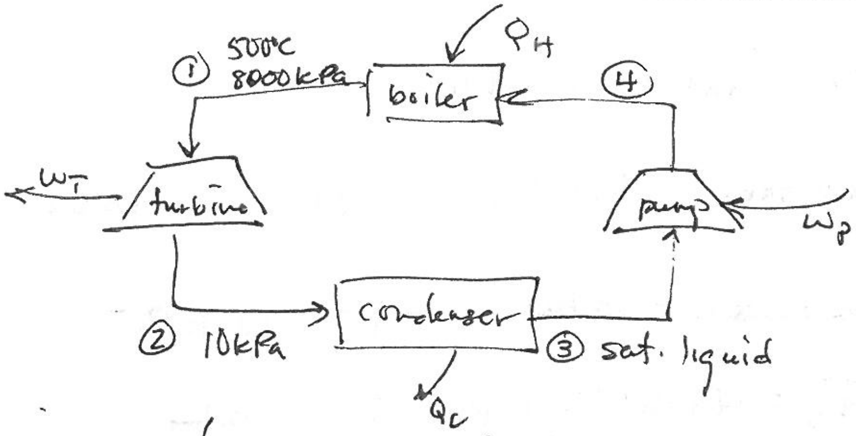
Power Cycles
Example 1

Steam generated in a power plant at a pressure of 8000 kPa and a temperature of 500 °C is fed to a turbine. Exhaust from the turbine enters a condenser at 10 kPa, where it is condensed to saturated liquid, which is then pumped to the boiler.

a) What is the thermal efficiency of a Rankine cycle operating at these conditions?

b) What is the thermal efficiency of a practical cycle operating at these conditions if the turbine efficiency and pump efficiency are both 0.75?

c) If the rating of the power cycle of part (b) is 80,000 kW, what is the steam rate and what are the heat-transfer rates in the boiler and condenser?



	T	P	$\hat{H}$ (kJ/kg)	$\hat{S}$ (kJ/kg·K)
1	500°C	8000 kPa	3398.3	6.724
2	45.81	10 kPa	2130.1	6.724
t. liq. 3	45.81	10 kPa	191.83	
4			199.9	

We can get $\hat{H}_1$ & $\hat{S}_1$ from Steam tables (Values in table)

Entropy Bal around turbine

ote: Rankine assumes reversible

$$\cancel{\frac{dS}{dt}}^{0} = \sum \dot{m}_k \hat{S}_k + \cancel{\frac{\dot{Q}_c}{T}}^{0} + \cancel{\dot{S}_{gen}}^{0} \Longrightarrow \hat{S}_1 = \hat{S}_2$$

From steam tables at 10 kPa, it is seen that sat. vapor too high of $\hat{S}$ $\Longrightarrow$ ∴ must be mixture

$\hat{S}_2^L = 0.6493$

$\hat{S}_2^V = 8.1502$

$$\hat{S}_2 = (1-x)\hat{S}_2^L + x\hat{S}_2^V$$
$$= \hat{S}_2^L - x\hat{S}_2^L + x\hat{S}_2^V$$
$$= \hat{S}_2^L + x\left(\hat{S}_2^V - \hat{S}_2^L\right)$$

solve for $x = 0.810$

$\hat{H}_2^V = 2584.7$

$\hat{H}_2^L = 191.83$

$$\hat{H}_2 = (1-x)\hat{H}_2^L + x\hat{H}_2^V$$

$\hat{H}_2 = 2130.1$ (Put $\hat{H}_2$ in table)

Read T from table (Put $T_2 = 45.81$°C in table)

~~We know condenser isobaric ($P_3 = 10$ kPa)~~

~~∴ $T_3 = 45.81$°C (Put T_3 in table)~~

Calculate work of turbine

$$\cancel{\frac{dU}{dt}}^{0} = \sum \dot{m}_k \hat{H}_k + \cancel{\dot{Q}}^{0} + \dot{W}_T$$

$$\dot{W}_T = \dot{m}\left(\hat{H}_2 - \hat{H}_1\right)$$

$$\frac{\dot{W}_T}{\dot{m}} = \hat{H}_2 - \hat{H}_1 = -1268.2 \text{ kJ/kg}$$

We know condenser isobaric ($P_3 = 10$ kPa)

∴ $T_3 = 45.81$°C (Put T_3, P_3 in table)

Because it is sat. liquid $\hat{H}_3 = \hat{H}_3^L = 191.83$ (Put $\hat{H}_3$ in table)

Energy Bal around condenser,

$$\cancel{\frac{dU}{dt}}^{0} = \sum \dot{m}_k \hat{H}_k + \dot{Q}_C + \cancel{\dot{W}}^{0}$$

$$\dot{Q}_c = \dot{m}(\hat{H}_3 - \hat{H}_2)$$

$$\frac{\dot{Q}_c}{\dot{m}} = \hat{H}_3 - \hat{H}_2$$

$$= -1938.3 \text{ kJ/kg}$$

Pump is isentropic $(\hat{S}_3 = \hat{S}_4)$

$$\cancelto{0}{\frac{dU}{dt}} = \sum \dot{m}_k \hat{H}_k + \cancelto{0}{\dot{Q}} + \dot{W}_p - \cancelto{0}{\int p\,dV}$$

$$\frac{\dot{W}_p}{\dot{m}} = \hat{H}_4 - \hat{H}_3$$

We need to get $\hat{H}_4$ — Recall that typically we would solve entropy balance

↳ S is isentropic $\not\Rightarrow$ No info

you would see that you couldn't get $\hat{H}_4$ b/c subcooled

∴ We need to relate ΔH in terms of pressure

<u>Note</u>: That we can't get $\hat{H}_4$ from Steam Tables since it is subcooled

internal energy should increase

we know $dU = TdS - pdV$ relates S to PdV

$$H = U + PV$$

or $$dH = dU + PdV + Vdp$$

combine $$dH = T\cancel{dS} + Vdp$$

isentropic

$$\therefore \boxed{\frac{\dot{W}_p}{\dot{m}} = \int \hat{V}dp}$$ incompressible → important equation

$$= \hat{V}(P_4 - P_3)$$

Get $\hat{V}$ from steam tables $\hat{V}^L = 1.01 \times 10^{-3}$ m³/kg

$$\frac{\dot W_P}{\dot m} = \left(1.01\times10^{-3}\ m^3/kg\right)\left(8000 - 10\ kPa\right)$$

$$= 8.07\ KJ/kg$$

$$\frac{\dot W_P}{\dot m} = \hat H_4 - \hat H_3 \quad \text{or} \quad \hat H_4 = \frac{\dot W_P}{\dot m} + \hat H_3$$

$\hat H_4 = 199.9$ (Put $\hat H_4$ in table)

Energy Balance around boiler

$$\cancelto{0}{\frac{dU}{dt}} = \sum \dot m_k \hat H_k + \dot Q_H + \cancelto{0}{\dot W}$$

$$\frac{\dot Q_H}{\dot m} = \hat H_1 - \hat H_4$$

$$\frac{\dot Q_H}{\dot m} = 3198.4\ KJ/kg$$

W_S (Rankine) = Net work $= \frac{\dot W_T}{\dot m} + \frac{\dot W_P}{\dot m}$

$$= -1268.2 + 8.07\ KJ/kg$$

$$= -1260.1\ KJ/kg$$

Note $W_S = -(Q_H + Q_c)$

$$= -(3198.4 - 1938.3)$$

$$= -1260.1\ KJ/kg$$

check on work

$$\eta = \frac{-W_S}{Q_H} = \frac{+1260.1\ KJ/kg}{3198.4\ KJ/kg} \Rightarrow \boxed{\eta = 0.394}$$

(b) $\eta_T = \eta_P = 0.75$

$$\eta_T = \frac{W_s}{W_s\,(\text{isentropic})} = \frac{\Delta\hat{H}}{(\Delta\hat{H})_s}$$

$$\hat{\Delta} H = \eta_T (\Delta\hat{H})_s \quad \text{from part (a)}$$

$$= 0.75(-1268.2)$$

$$= -951.1 \text{ kJ/kg}$$

	T	P	H	S
1	500	8000	3398.3	6.724
2	45.81	10	2447.2	7.7
3				
4				

$$\hat{H}_2' = \hat{H}_1 + \Delta\hat{H}$$

$$= 3398.3 - 957.1$$

$$= 2447.2 \quad \text{(Put in table)}$$

We see from steam tables that state 2 is also wet

$$\hat{H}_2' = \hat{H}_2^L + x(\hat{H}_2^V - \hat{H}_2^L)$$

$$2447.2 = 191.83 + x\,(2584.7 - 191.83)$$

$$x = 0.94$$

$$\hat{S}_2 = \hat{S}_2^L + x(\hat{S}_2^V - \hat{S}_2^L)$$

$$= 7.7 \quad \left(\text{Put in table } S_2 \;\&\; T_2\right)$$

$$\frac{\dot{W}_T}{\dot{m}} = \hat{H}_2' - \hat{H}_1$$

$$= -951.1 \text{ kJ/kg}$$

$$\frac{\dot{Q}_c}{\dot{m}} = \hat{H}_3 - \hat{H}_2'$$

$$= 191.83 - 2447.2$$

$$= -2255.4$$

$$\eta_{pump} = \text{[illegible]} \qquad \frac{(\Delta H)_s}{\Delta H}$$

$$\Delta\hat{H} = \cancel{0.75\,(\Delta\hat{H})_s} \quad = \frac{8.07}{0.75}$$

$$= \cancel{0.75\,(8.07)}$$

$$= 10.76 \text{ kJ/kg}$$

~~W_s (Rankine) =~~ $\frac{\dot{W}_p}{\dot{m}} = 10.76$ kJ/kg

$$W_s \text{ (Rankine)} = -951.1 + 10.76$$

$$= -940.3 \text{ kJ/kg}$$

$$\hat{H}_4 = \hat{H}_3 + \Delta\hat{H}$$

$$= 191.83 + 10.76$$

$$= 202.59 \text{ kJ/kg}$$

$$\frac{\dot{Q}_H}{\dot{m}} = \hat{H}_1 - \hat{H}_4 \quad = 339.8.3 - 202.59$$

$$= 3195.71$$

$$\eta = \frac{-W_S}{Q_H} = \frac{940.3}{3195.71} = 0.294$$

(c) $\dot{W}_S = 80000 \text{ kW}$

$$\dot{W}_S = \dot{m} W_S$$

$$\dot{m} = \frac{\dot{W}_S}{W_S}$$

$$= \frac{80000 \text{ kJ/s}}{940.3 \text{ kJ/kg}}$$

$$\dot{m} = 85.1 \text{ kg/s}$$

$$\dot{Q}_H = \dot{m} Q_H$$

$$= (85.1)(3195.71)$$

$$= 2.7 \times 10^5 \text{ kJ/s}$$

$$\dot{Q}_c = \dot{m} Q_c$$

$$= (85.1)(-2255.4)$$

$$= -1.9 \times 10^5 \text{ kJ/s}$$